CHAPTER

8 CIRCLES

Syllabus

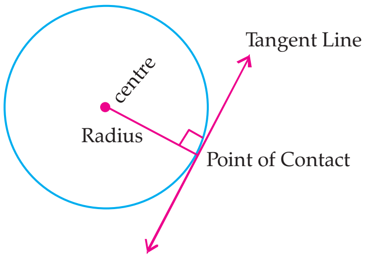
- Tangent to a circle at point of contact.
  1. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact.
  2. (Prove) The lengths of tangents drawn from an external point to a circle are equal.

Trend Analysis

List of Concepts	2018		2019		2020	
	Delhi	Outside Delhi	Delhi	Outside Delhi	Delhi	Outside Delhi
Tangent to Circle (Theorems)	1 Q (3 M)			1 Q (3 M)	1 Q (2 M) 2 Q (3 M)	1 Q (3 M)
Question based on Properties of tangent			2 Q (3 M)		1 Q (1 M)	1 Q (1 M)

Revision Notes

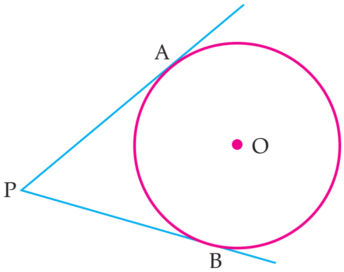
- A tangent to a circle is a line that intersects the circle at one point only.
- The common point of the circle and the tangent is called the point of contact.
- **Secant:** Two common points (A and B) between line PQ and circle.
- A tangent to a circle is a special case of the secant when the two end points of the corresponding chord are coincide.
- There is no tangent to a circle passing through a point lying inside the circle.
- At any point on the circle there can be one and only one tangent.
- The tangent at any point of a circle is perpendicular to the radius through the point of contact.



- There are exactly two tangents to a circle through a point outside the circle.
- The length of the segment of the tangent from the external point P and the point of contact with the circle is called the length of the tangent.
- The lengths of the tangents drawn from an external point to a circle are equal.

  In the figure,

$$PA = PB.$$

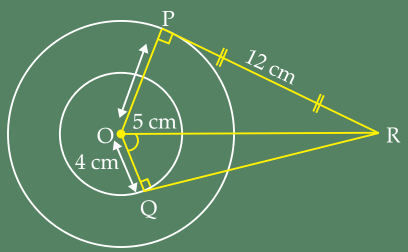


Know the Facts

- The word 'tangent' comes from the Latin word 'tangere', which means to touch and was introduced by the Danish mathematician **Thomas Fincke** in 1583.
- The line perpendicular to the tangent and passing through the point of contact, is known as the normal.
- In two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

How is it done on the GREENBOARD?

Q.1. In the given figure if $PR = 12$ cm, $OP = 5$ cm and $OQ = 4$ cm, find RQ.



Solution:

Step I: $OP \perp RP$ (as radius is $\perp$ to tangent at point of contact)

In right triangle OPR,

$$OR^2 = OR^2 + OR^2$$

$$OR^2 = 5^2 + 12^2$$

$$OR^2 = 25 + 144$$

$$OR^2 = 169$$

or, $OR = \sqrt{169} = 13$ cm

Step II: Similarly, in right triangle OQR,

$$OR^2 = OQ^2 + QR^2$$

$$QR^2 = OR^2 - OQ^2$$

$$QR^2 = 13^2 - 4^2$$

or, $QR^2 = 169 - 16$

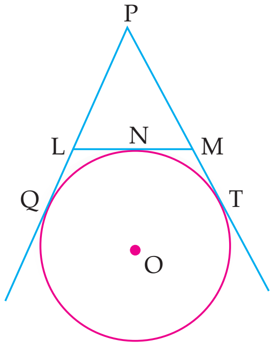
$$QR^2 = 153$$

or, $QR = \sqrt{153}$ cm.

$= 3\sqrt{17}$ cm

Very Short Answer Type Questions — 1 mark each

Q. 1. If $PQ = 28$ cm, then find the perimeter of ΔPLM.



A [CBSE SQP, 2020-21]

Sol. $\because$ $PQ = PT$

$PL + LQ = PM + MT$

$PL + LN = PM + MN$

$(LQ = LN, MT = MN)$

(Tangents to a circle from a common point)

Perimeter (ΔPLM)

$= PL + LM + PM$ ½

$= PL + LN + MN + PM$

$= 2(PL + LN)$

$= 2(PL + LQ)$

$= 2 \times 28 = 56$ cm ½

[CBSE SQP Marking Scheme, 2020-21]

Detailed Solution:

Given, $PQ = 28$ cm

$\therefore$ $PQ = PT$

(Length of tangents from an external point are equal)

i.e., $PQ = PT = 28$ cm

According to figure,

Let $LQ = x$, then

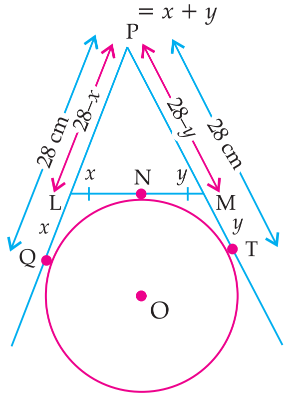
$PL = (28 - x)$ cm

and let $MT = y$, then

$PM = (28 - y)$ cm

and $LM = LN + NM$

$= x + y$



½

Now, the perimeter of $\Delta PLM = PL + LM + PM$

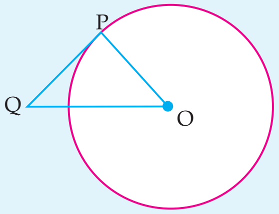
$= (28 - x) + (x + y) + (28 - y)$

$= 28 + 28 = 56$ cm. ½

AI **Q. 2. PQ is a tangent to a circle with centre O at point P. If ΔOPQ is an isosceles triangle, then find $\angle OQP$.**

A [CBSE SQP, 2020-21]

Sol.



In ΔOPQ,

$\angle P + \angle Q + \angle O = 180°$

$(\angle O = \angle Q$ isosceles triangle)

$2\angle Q + \angle P = 180°$

$2\angle Q + 90° = 180°$

$2\angle Q = 90°$

$\angle Q = 45°$ 1

[CBSE SQP Marking Scheme, 2020-21]

Detailed Solution:

As we know that

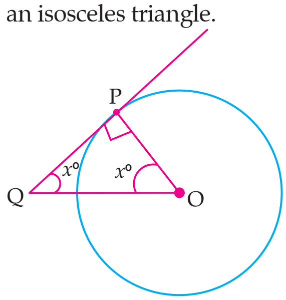
$\angle OPQ = 90°$ (Angle between tangent and radius)

Let ∠PQO be $x°$, then

$\angle QOP = x°$

Since OPQ is an isosceles triangle. (given)

(OP = OQ)



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In ΔOPQ,

$\angle OPQ + \angle PQO + \angle QOP = 180°$ (Property of the sum of angles of a triangle)

$\therefore$ $90° + x° + x° = 180°$

$\Rightarrow$ $2x° = 180° - 90° = 90°$

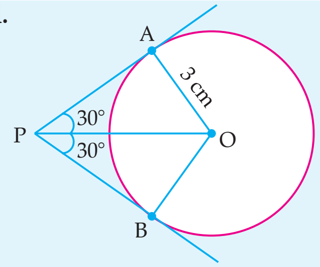
$\Rightarrow$ $x = \frac{90}{2} = 45°.$

Hence, ∠OQP is 45° ½

Q. 3. If two tangents inclined at 60° are drawn to a circle of radius 3 cm, then find length of each tangent.

A [CBSE SQP, 2020-21]

Sol.



In ΔPAO,

$$\tan 30° = \frac{AO}{PA}$$

(Using trigonometry) ½

$$\frac{1}{\sqrt{3}} = \frac{3}{PA}$$

$$PA = 3\sqrt{3} \text{ cm.}$$ ½

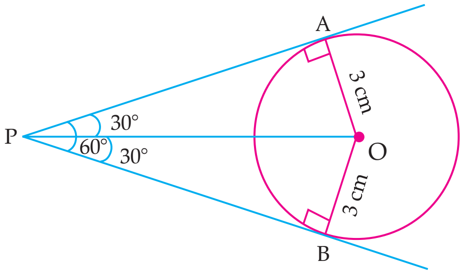
[CBSE SQP Marking Scheme, 2020-21]

Detailed Solution:

$$PA = PB = ?$$

Angle between tangents $= 60°$ (Given)

$\therefore$ Tangents are equally inclined to each other.



½

$\Rightarrow$ $\angle OPA = \angle OPB = 30°$

and $\angle OAP = 90°$

(Angle between tangent and radius)

In ΔPAO,

$$\tan 30° = \frac{\text{Perpendicular}}{\text{Base}} = \frac{OA}{AP}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = 3\sqrt{3}$$

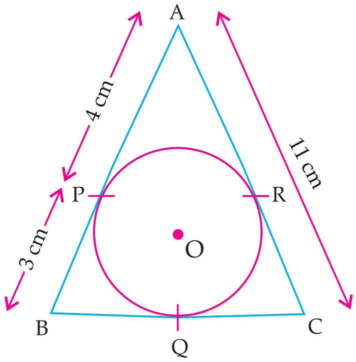
(Using trigonometric Ratios)

$$\Rightarrow \quad AP = 3\sqrt{3}$$

Hence, the length of each tangent is $3\sqrt{3}$ cm. ½

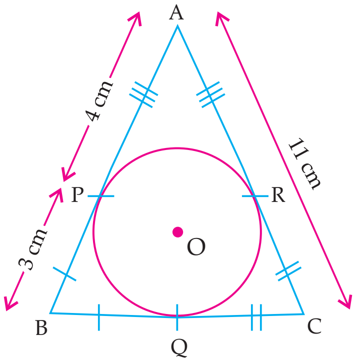
AI Q. 4. In the adjoining figure, if ΔABC is circumscribing a circle, then find the length of BC.

U **[CBSE Delhi Set-I, 2020]**



Sol. $\because$ AP and AR are tangents to the circle from external point A.

$\therefore$ $AP = AR$ *i.e.*, $AR = 4$ cm



Similarly, PB and BQ are tangents.

$\therefore$ $BP = BQ$ *i.e.*, $BQ = 3$ cm ½

Now, $CR = AC - AR = 11 - 4 = 7$ cm

Similarly, CR and CQ are tangents.

$\therefore$ $CR = CQ$ *i.e.*, $CQ = 7$ cm

Now, $BC = BQ + CQ = 3 + 7 = 10$ cm.

Hence, the length of BC is 10 cm. ½

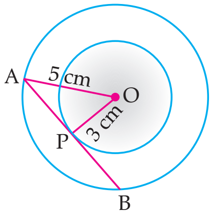
COMMONLY MADE ERROR

- Some students were not versed with the properties of circle.

ANSWERING TIP

- It is necessary for the students to learn all properties of circle.

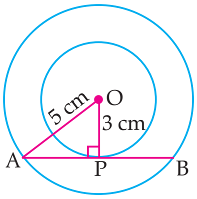
AI Q. 5. In the given figure, find the length of PB.



U **[CBSE OD Set-I, 2020]**

Sol. Since AB is a tangent at P and OP is radius.

$\therefore$ $\angle APO = 90°$, $AO = 5$ cm and $OP = 3$ cm



In right angled ΔOPA,

$$AP^2 = AO^2 - OP^2$$

(By using Pythagoras theorem) ½

$$AP^2 = (5)^2 - (3)^2 = 25 - 9 = 16$$

$\Rightarrow$ $AP = 4$ cm

$\because$ Perpendicular from centre to chord bisect the chord

$\Rightarrow$ $AP = BP = 4$ cm. ½

Q. 6. If the radii of two concentric circles are 4 cm and 5 cm, then find the length of each chord of one circle which is tangent to the other circle.

[CBSE SQP, 2020]

Sol. Length of Tangent $= 2 \times \sqrt{5^2 - 4^2}$

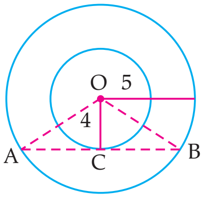
$= 2 \times 3 \text{ cm} = 6 \text{ cm}$ ½+½

[CBSE SQP Marking Scheme, 2020]

Detailed Solution:

In ΔOBC

$$CO^2 + BC^2 = OB^2$$
$$4^2 + BC^2 = 5^2$$
$$16 + BC^2 = 25$$
$$BC^2 = 25 - 16$$
$$BC^2 = 9$$
$$BC = 3$$



½

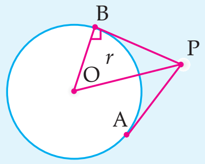
In ΔOAC,

$$OC^2 + AC^2 = OA^2$$
$$4^2 + AC^2 = 5^2$$
$$AC^2 = 9$$
$$AC = 3$$
$$\therefore \quad AB = AC + BC$$
$$= 3 + 3$$
$$= 6 \text{ cm.}$$ ½

AI **Q. 7. If the angle between two tangents drawn from an external point 'P' to a circle of radius 'r' and centre O is 60°, then find the length of OP.**

[CBSE SQP, 2020]
[Foreign Set-I, II, III, 2016]

Sol.



In OBP, $\dfrac{OB}{OP} = \sin 30°$ ½

$\therefore \quad OP = 2r$ ½

[CBSE Marking Scheme, 2020]

Detailed Solution:

$$OA = r$$
$$PP = ?$$

Angle between tangents $= 60°$

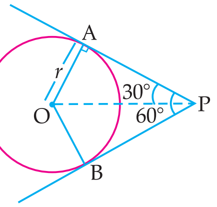
Tangents are equally inclined to each other

$\Rightarrow \quad \angle OPA = \angle OPB = 30°$

In ΔOPA,

$$\angle POA = 180° - 90° - 30°$$
$$= 60°$$ ½

$\because \quad \cos 60° = \dfrac{OA}{OP}$



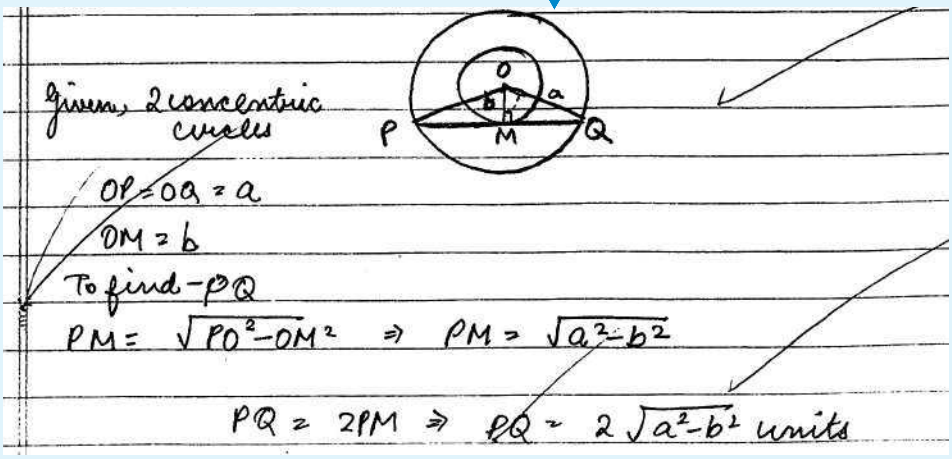
$$\frac{1}{2} = \frac{r}{OP}$$

$\Rightarrow \quad OP = 2r.$ ½

Q. 8. Two concentric circles of radii a and b ($a > b$) are given. Find the length of the chord of the larger circle which touches the smaller circle. [CBSE, Delhi Region, 2019]

Topper Answer, 2019

Sol.



Given, 2 concentric circles

$OP = OQ = a$

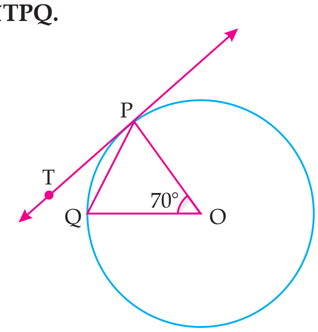
$OM = b$

To find – PQ

$PM = \sqrt{PO^2 - OM^2} \Rightarrow PM = \sqrt{a^2 - b^2}$

$PQ = 2PM \Rightarrow PQ = 2\sqrt{a^2 - b^2}$ units

Q. 9. **In given figure, O is the centre of the circle, PQ is a chord and PT is tangent to the circle at P. Find ∠TPQ.**



U **[CBSE, OD Set-I, II, III, 2017]**

Sol. $\angle OPQ = \angle OQP$ (radius of circle)

$= \dfrac{180° - 70°}{2} = 55°$ ½

∴ $\angle TPQ = 90° - 55°$

$= 35°$ ½

[CBSE Marking Scheme, 2017]

Detailed Solution:

According to the figure,

$OP = OQ$ (radii)

∴ $\angle OPQ = \angle OQP$

(Isosceles triangle property)

Now, in ΔPOQ,

$\angle OPQ + \angle OQP + \angle POQ = 180°$

(Angle sum property)

$\angle OPQ + \angle OPQ + 70° = 180°$

$\angle OPQ = 180° - 70° = 110°$ ½

$\angle OPQ = 55°$

Since $\angle OPT = 90°$ (Angle between tangent and radius)

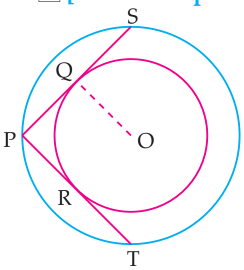
Hence, $\angle TPQ = 90° - \angle OPQ$

$= 90° - 55°$

$= 35°.$ ½

Q. 10. **In the fig. there are two concentric circles with centre O. PRT and PQS are tangents to the inner circle from a point P lying on the outer circle. If PR = 5 cm, find the length of PS.**

U **[Delhi Comptt. Set-I, II, III, 2017]**



Sol. $PQ = PR = 5$ cm ½

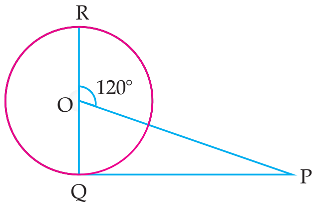
and $PQ = QS$ ½

∴ $PS = 2PQ$

$= 2 \times 5 = 10$ cm.

[CBSE Marking Scheme, 2017]

Q. 11. **PQ is a tangent drawn from an external point P to a circle with centre O and QOR is the diameter of the circle. If $\angle POR$ = 120°, What is the measure of ∠OPQ ?** U **[Foreign Set-I, II, III, 2016, 2017]**



Sol. In ΔOQP $\angle POR = \angle OQP + \angle OPQ$

(Exterior angle) ½

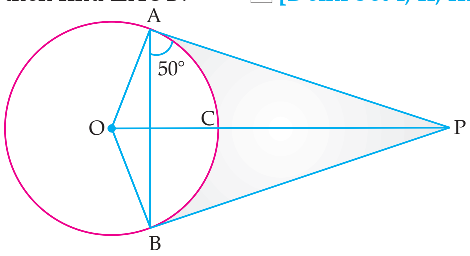
∴ $\angle OPQ = \angle POR - \angle OQP$

$= 120° - 90$

$= 30°$ ½

Q. 12. **From an external point P, tangents PA and PB are drawn to a circle with centre O. If $\angle PAB$ = 50°, then find ∠AOB.** U **[Delhi Set-I, II, III, 2016]**

Sol.



Here, $\angle OAB = 90° - 50°$

$= 40°$ (∵ PA ⊥ OA)

$\angle OAB = \angle OBA = 40°$

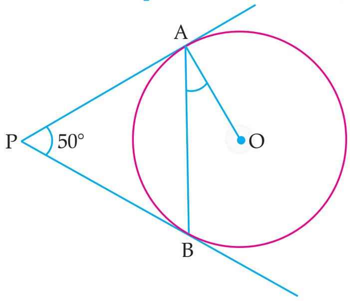
(∵ OA and OB are radii)

∴ $\angle AOB + 40° + 40° = 180°$

$\angle AOB = 180° - 80° = 100°$

Hence $\angle AOB = 100°$ 1

Q. 13. **In fig., PA and PB are tangents to the circle with centre O such that $\angle APB$ = 50°. Write the measure of ∠OAB.** A **[CBSE, Delhi Set I, II, III, 2015]**



Sol. Here, $\angle APB = 50°$

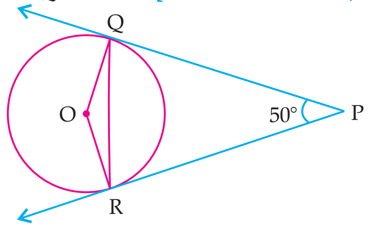
$\angle PAB = \angle PBA = \dfrac{180° - 50°}{2} = 65°$

$\angle OAB = 90° - \angle PAB$

$= 90° - 65° = 25°$

[CBSE Marking Scheme, 2015] 1

Q. 14. **In the given figure, PQ and PR are tangents to the circle with centre O such that $\angle QPR$ = 50°, then find ∠OQR.** **[CBSE Delhi Set-I, II, III, 2015]**



Sol. $\angle QPR = \angle 50°$ (Given)

$\angle QOR + \angle QPR = 180°$

(Supplementary angles)

$\therefore$ $\angle QOR = 180° - 50° = 130°$ ½

From ΔOQR,

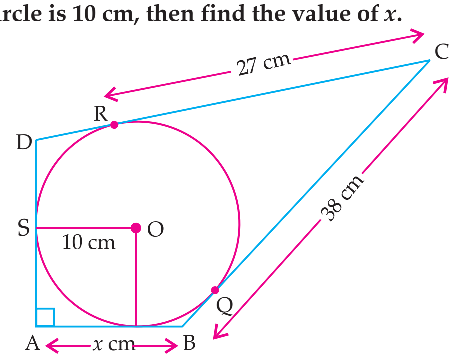
or, $\angle OQR = \angle ORQ = \dfrac{180° - 130°}{2}$

$= \dfrac{50°}{2} = 25°$ ½

[CBSE Marking Scheme, 2015]

Short Answer Type Questions-I — 2 marks each

Q. 1. In the figure, quadrilateral ABCD is circumscribing a circle with centre O and AD ⊥ AB. If radius of incircle is 10 cm, then find the value of *x*.



A [CBSE SQP, 2020-21]

Sol. $\angle A = \angle OPA = \angle OSA = 90°$ ½

Hence, $\angle SOP = 90°$

Also, $AP = AS$

Hence, OSAP is a square.

$AP = AS = 10$ cm ½

$CR = CQ = 27$ cm

$BQ = BC - CQ$

$= 38 - 27 = 11$ cm ½

$BP = BQ = 11$ cm

$x = AB = AP + BP$

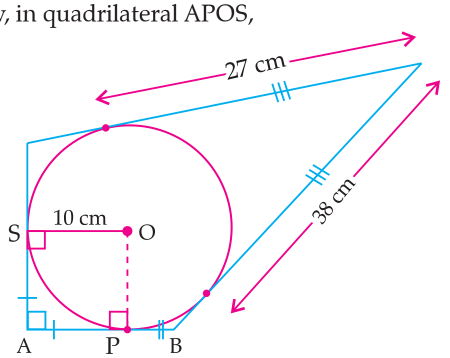
$= 10 + 11 = 21$ cm ½

[CBSE Marking Scheme, 2020-21]

Detailed Solution:

With O as centre, draw a perpendicular OP on AB.

Now, in quadrilateral APOS,



$\angle SAP = 90°$ (Given)

$\angle APO = 90°$ (By construction)

and $\angle ASO = 90°$

(Angle between tangent and radius)

Finally $\angle SOP = 360° - (90° + 90° + 90°) = 90°$

$AP = AS$

(Tangents from external point A)

$\therefore$ *OSAP* is a square.

$AP = AS = SO = 10$ cm ½

$\because$ $CR = CQ$

(Tangents from external point C)

$\therefore$ $CR = CQ = 27$ cm

But $BC = 38$ cm (Given)

$\therefore$ $BQ = BC - CQ = (38 - 27)$ cm

$BQ = 11$ cm ½

$BP = BQ$

(Tangent from external point B)

$\therefore$ $BP = 11$ cm

So, $x = AB = AP + PB$

$= (10 + 11)$ cm $= 21$ cm

Hence, the value of x is 21 cm. ½

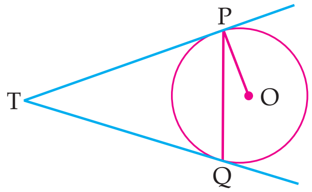
COMMONLY MADE ERROR

- Some students does not use appropriate figure to solve the question.

ANSWERING TIP

- Carefully read the question and draw the figure as per the required condition.

AI **Q. 2. In the given figure, two tangents TP and TQ are drawn to circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.**



U [CBSE Delhi Set-I, 2020]

[CBSE Delhi Set-I, II, III, 2017]

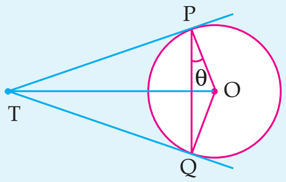
Sol. Let $\angle OPQ$ be θ, then

$\angle TPQ = 90° - \theta$ ½

Since, $TP = TQ$

$\therefore$ $\angle TQP = 90° - \theta$ ½

(opposite angles of equal sides)



Now, $\angle TPQ + \angle TQP + \angle PTQ = 180°$ ½

(Angle sum property of a Triangle)

$\Rightarrow\ 90° - \theta + 90° - \theta + \angle PTQ = 180°$

$\Rightarrow\ \angle PTQ = 180° - 180° + 2\theta$

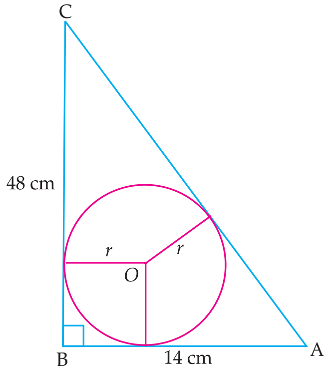
$\Rightarrow\ \angle PTQ = 2\theta$

Hence, $\angle PTQ = 2\angle OPQ$ ½

Hence Proved.

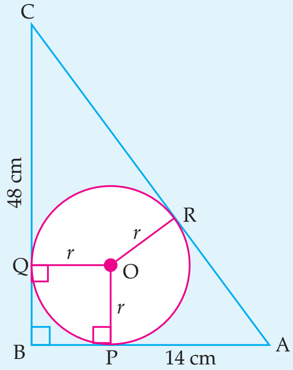
[CBSE Marking Scheme, 2020]

Q. 3. In Fig, ABC is a triangle in which $\angle B = 90°$, BC = 48 cm and AB = 14 cm. A circle is inscribed in the triangle, whose centre is O. Find radius of incircle.



A **[CBSE Comptt. Set I, II, III, 2018]**

Sol. $AC = \sqrt{AB^2 + BC^2}$



$= \sqrt{14^2 + 48^2} = \sqrt{2500} = 50$ cm ½

$\angle OQB = 90° \Rightarrow OPBQ$ is a square

$BQ = r$, $QC = 48 - r = CR$ ½

Again, $PB = r$

$PA = 14 - r \Rightarrow RA = 48 - r$ ½

$AR + RC = AC \Rightarrow 14 - r + 48 - r = 50$

$r = 6$ cm ½

[CBSE Marking Scheme, 2018]

Detailed Solution:

In ΔABC, $\angle B = 90°$ (Given)

$AC^2 = AB^2 + BC^2$

(By using pythagoras theorem)

$= (14)^2 + (48)^2 = 196 + 2304$

$= 2500$

$\therefore\ AC = \sqrt{2500} = 50$ cm ½

Here, $\angle OQB = \angle OPB = 90°$

(Radius is perpendicular to tangent)

$\therefore$ In Quadrilateral OPBQ,

$\angle POQ = 360° - (OQB + \angle OPB + \angle PBQ)$

$= 360° - (90° + 90° + 90°) = 90°$

So, $OPBQ$ is a square.

Then $OP = QB = BP = OQ = r$ ½

Thus, $CQ = BC - QB = 48 - r$

But $CQ = CR$

(Tangents from external point C)

$\therefore\ CR = 48 - r$

and $AP = AB - BP = 14 - r$ ½

But $AP = AR$

(Tangents from external point A)

$\therefore\ AR = 14 - r$

Now $AC = 50$ cm (proved above)

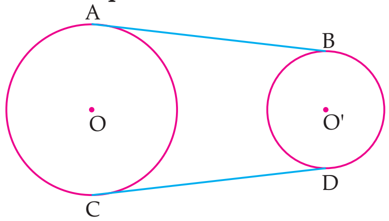
$\Rightarrow\ AR + RC = 50$

$\Rightarrow\ 14 - r + 48 - r = 50$

$\Rightarrow\ -2r = 50 - 62 = -12$

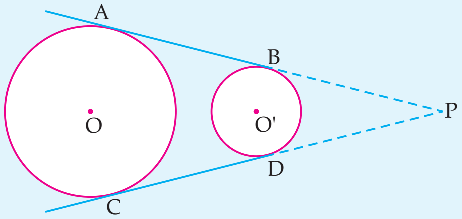
$\Rightarrow\ r = 6$ cm. ½

Q. 4. In the fig, AB and CD are common tangents to two circles of unequal radii. Prove that $AB = CD$.



A **[Delhi Comptt. Set-III, 2017]**

Sol. Construction : Produce AB and CD to meet at P.



Now, $PA = PC$

and $PB = PD$ ½

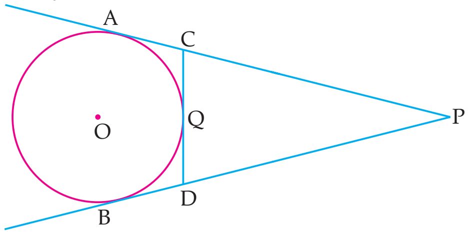
Tangents to a circle from external point ½

Now, $PA - PB = PC - PD$ ½

$\Rightarrow\ AB = CD$ ½

[CBSE Marking Scheme, 2017]

Q. 5. In the given figure, PA and PB are tangents to the circle from an external point P. CD is another tangent touching the circle at Q. If PA = 12 cm, QC = DQ = 3 cm, then find $PC + PD$.



A **[Delhi Comptt. Set-I, II, III, 2017]**

Sol. Here, $AC = CQ$ (Tangents from external point to a circle)

$PA = PC + CA = PC + CQ$ $(\because\ CA = CQ)$

$\Rightarrow\quad 12 = PC + 3$

$\Rightarrow\quad PC = 12 - 3 = 9$ cm **1**

$PB = PD + BD$

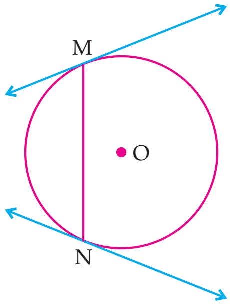
$PA = PD + DQ$

$12 - 3 = PD = 9$ cm

$\therefore\quad PC + PD = 9 + 9 = 18$ cm **1**

[CBSE Marking Scheme, 2017]

Q. 6. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.



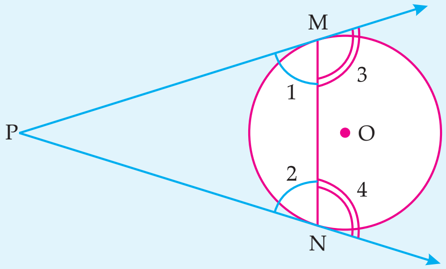
A **[CBSE, OD Set-I, II, III, 2017] [CBSE Delhi Term-II, 2015]**

Sol. $\because$ PM = PN (length of tangents are equal)

$\angle 1 = \angle 2$ (angles opp. to equal sides are equal) **1**

$\because\quad 180° - \angle 1 = 180° - \angle 2$ (linear pair)

$\angle 3 = \angle 4$ **1**

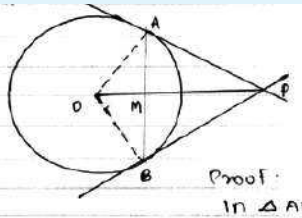


[CBSE Marking Scheme, 2015]

Detailed Solution:

Topper Answer, 2017

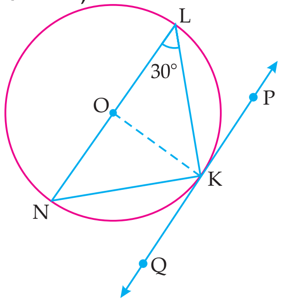
Sol.



Given: chord AB.
tangents AP and BP at A & B.
To prove: ~~AP = BP~~ ∠PAM = ∠PBM
Construction: Join centre O to P
let OP meet AB at M.
Proof:
In △AMP and △BMP,
AP = BP — tangents from same point to a circle are equal.
MP = MP — common side
∠APM = ∠BPM — tangents are equally inclined to line joining the point of emergence to circle's centre.
by SAS criterion,
△AMP ≅ △BMP.
by cpct. ∠PAM = ∠PBM
Hence, tangents at end points of a chord make equal angles with it

2

Q. 7. **In given figure, O is the centre of the circle and LN is a diameter. If PQ is a tangent to the circle at *K* and $\angle KLN = 30°$, find $\angle PKL$.**



U [CBSE OD Comptt. Set-I, II, III, 2017]

Sol. Here, $OK = OL$ (radii)

$\angle OKL = \angle OLK = 30°$

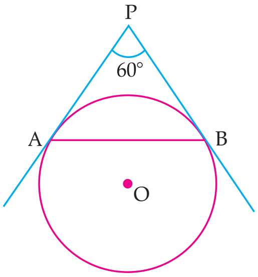
(Opposite angles of equal sides) **1**

Since $\angle OKP = 90°$ (Tangent)

$\therefore$ $\angle PKL = 90° - 30° = 60°$ **1**

[CBSE Marking Scheme, 2017]

Q. 8. **In Fig., AP and BP are tangents to a circle with centre O, such that *AP* = 5 cm and $\angle APB = 60°$. Find the length of chord *AB*.**



U [CBSE Delhi Set I, II, III, 2016]

Sol. $PA = PB$ ½

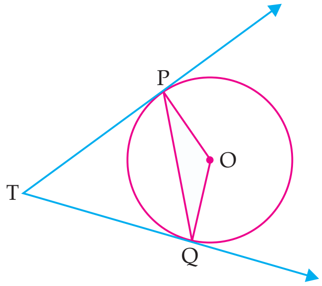
or, $\angle PAB = \angle PBA = 60°$ ½

$\therefore$ ΔPAB is an equilateral triangle. ½

Hence, $AB = PA = 5$ cm. ½

[CBSE Marking Scheme, 2016]

Q. 9. **In the given figure PQ is chord of length 6 cm of the circle of radius 6 cm. TP and TQ are tangents to the circle at points P and Q respectively. Find $\angle PTQ$.**



U [CBSE S.A.II, 2016]

Sol. Here, $PQ = 6$ cm, $OP = OQ = 6$ cm

$\therefore$ $PQ = OP = OQ$

$\therefore$ $\angle POQ = 60°$

(angle of equilateral Δ) ½

$\angle OPT = \angle OQT = 90°$

(radius $\perp$ tangent)

$\therefore$ $\angle PTQ + 90° + 90° + 60° = 360°$

(angle sum property) ½

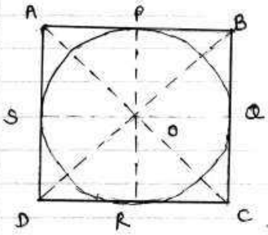
$\angle PTQ = 120°$ **1**

Q. 10. **A circle touches all the four sides of a quadrilateral ABCD. Prove that $AB + CD = BC + DA$.**

A [CBSE OD Set-I, II, III, 2016]

Topper Answer, 2016

Sol.



Given : circle touching sides of ABCD at P, Q, R & S.

To prove: AB + CD = AD + BC.

Proof :

AP = AS
PB = BQ
DR = DS
CR = CQ

} tangents from same point to a circle are equal in length

adding all (4),

AP + PB + DR + CR = AS + BQ + DS + CQ

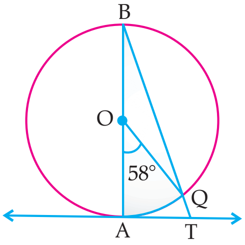
AB + CD = AS + SD + BQ + QC

AB + CD = AD + BC

Hence, proved.

2

Q. 11. In given figure, AB is the diameter of a circle with center O and AT is a tangent. If $\angle AOQ = 58°$, find $\angle ATQ$. U [Board Term-II, 2015 Set-I, II, III]



Sol. $\angle AOQ = 58°$ (Given)

$\angle ABQ = \frac{1}{2}\angle AOQ$

(Angle on the circumference of the circle by the same arc)

$= \frac{1}{2} \times 58°$

$= 29°$ **1**

$\angle BAT = 90°$ $(\because OA \perp AT)$

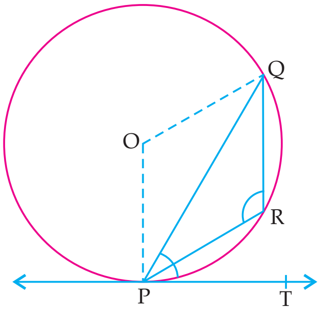
$\therefore$ $\angle ATQ = 90° - 29°$

$= 61°$ **1**

[CBSE Marking Scheme, 2015]

Q. 12. In figure, PQ is a chord of a circle centre O and PT is a tangent. If $\angle QPT = 60°$, find $\angle PRQ$.

[CBSE, OD Set-I, II, III, 2015]



Sol. Given, $\angle QPT = 60°$

$\angle OPQ = \angle OQP = 90° - 60° = 30°$

$\angle POQ = 180° - (30° + 30°)$

$= 180° - 60° = 120°$

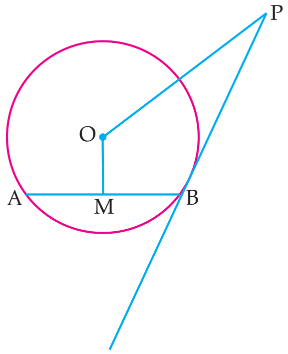
$\angle PRQ = \frac{1}{2}$ Reflex $\angle POQ$ **1**

$[\because$ Reflex $\angle POQ = 360° - 120° = 240°]$

$= \frac{1}{2} \times 240° = 120°$

[CBSE Marking Scheme, 2015] **1**

Q. 13. PB is a tangent to the circle with centre O to B. AB is a chord of length 24 cm at a distance of 5 cm from the centre. If the tangent is of length 20 cm, find the length of PO.



A [Delhi Board Term-2, 2015]

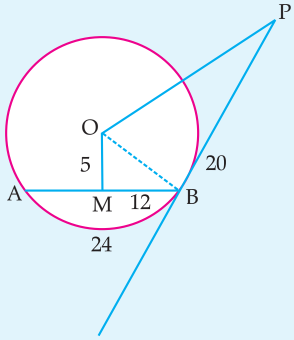
Sol. Construction : Join OB.

In rt. ΔOMB,

$OB^2 = 5^2 + 12^2 = 13^2$

$\therefore$ $OB = 13$ cm **1**

Since OB ⊥ PB (radius ⊥ tangent)



$\therefore$ In rt. ΔOBP,

$OP^2 = OB^2 + BP^2$

$= 13^2 + 20^2$

$= 569$

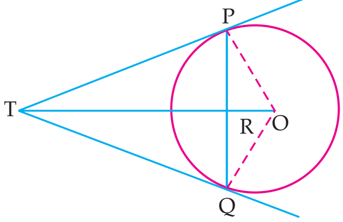
or, $OP = \sqrt{569} = 23.85$ cm. **1**

[CBSE Marking Scheme, 2015]

Q. 14. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of line segment PQ.

A [CBSE Delhi Term-2, 2015 Set-I, II, III]

Sol. Given: *A* circle with centre *O*. Tangents *TP* and *TQ* are drawn from a point *T* outside a circle.



To Prove: *OT* is the right bisector of line segment *PQ*.

Construction: Join OP and OQ

Proof: ΔOPT and ΔOTQ

$PT = PQ$ (Tangents of the circle)

$OT = OT$ (Common side)

$\angle OPT = \angle OQR = 90°$

$\therefore \quad \Delta OPT \cong \Delta OQT$ (R.H.S. Congruency)

$\angle PTO = \angle QTO$ (C.P.C.T)

ΔPTR and ΔTRQ

$TP = TQ$ (Tangents of circle)

$TR = TR$ (Common)

$\Delta PTR \cong \Delta QTR$ (SAS congruency)

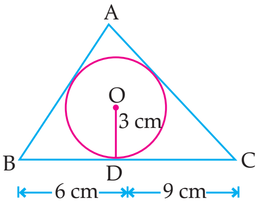
$\angle PRT = \angle TRQ$ (c.p.c.t.)

$PR = QR$ (c.p.c.t.)

$\angle PRT + \angle TRQ = 180°$

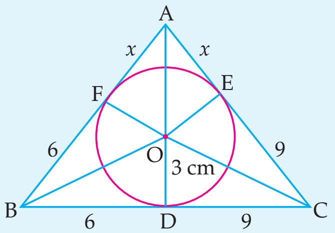
$\therefore \quad \angle PRT = \angle TRQ = 90°$

Q. 15. In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm², then find the lengths of sides AB and AC.



A [CBSE, OD Set-I, II, III, 2015]

Sol.



Let $AF = AE = x$.

$\therefore \quad AB = 6 + x, AC = 9 + x$ and $BC = 15$ ½

ar $\Delta ABC = \frac{1}{2}[15 + 6 + x + 9 + x].3 = 54$

$45 + 3x = 54$ **1**

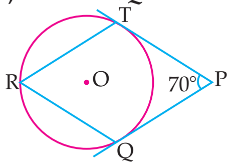
or, $x = 3$

$\therefore \quad AB = 9$ cm, $AC = 12$ cm ½

and $BC = 15$ cm.

[CBSE Marking Scheme, 2015]

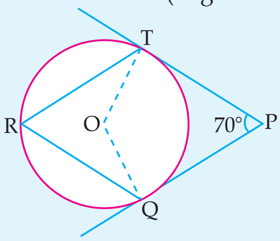
Q. 16. In figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If $\angle TPQ = 70°$, find $\angle$TRQ.



U [Foreign Set-I, II, III, 2015]

Sol. $\angle TOQ = 180° - 70° = 110°$ **1**

(angle of supplementary)



Then, $\angle TRQ = \frac{1}{2} \angle TOQ$

(angle at the circumference of the circle by same arc)

$= \frac{1}{2} \times 110° = 55°.$ **1**

[CBSE Marking Scheme, 2015]

Short Answer Type Questions-II — 3 marks each

Q. 1. Prove that the parallelogram circumscribing a circle is a rhombus. A [CBSE Delhi Set-II, 2020]

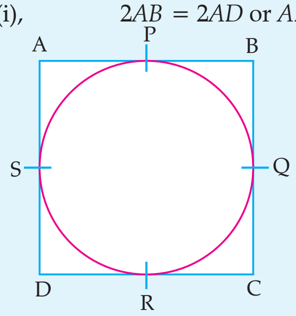
Sol. Let $ABCD$ be the || gm.

$\therefore \quad AB = CD$ and $AD = BC$ ...(i) ½

$AP + PB + DR + CR = AS + BQ + DS + CQ$ **1**

or, $AB + CD = AD + BC$ ½

From (i), $2AB = 2AD$ or $AB = AD$



or, $ABCD$ is a rhombus. **1**

[CBSE Marking Scheme, 2020]

Detailed Solution:

Let $ABCD$ be the parallelogram.

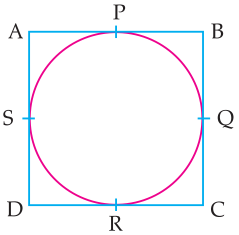
$\therefore AB = CD$ and $AD = BC$ ...(i) ½

We know that the tangents drawn to a circle from an exterior point are equal in length.

Therefore, $AP = AS$, $BP = BQ$, $CR = CQ$ and $DR = DS$. ½

Adding the above equations.

$AP + BP + CR + DR = AS + BQ + CQ + DS$ ½



½

$\Rightarrow (AP + BP) + (CR + DR) = (AS + DS)$
$+ (BQ + CQ)$

$\Rightarrow \qquad AB + CD = AD + BC$ ½

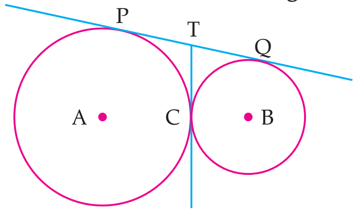
From eq. (i),

$$2AB = 2AD$$

or, $\qquad AB = AD$

Hence, $ABCD$ is a rhombus. **Hence Proved.** ½

AI **Q. 2. In given Fig., two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.**

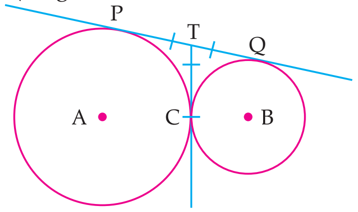


A **[CBSE Delhi Set-III, 2020]**

Sol. Since, $\qquad PT = TC \qquad$ (tangents of circle)

and $\qquad QT = TC$

(tangents of circle from extended point) **1**



So, $\qquad PT = QT$ ½

Now $\qquad PQ = PT + TQ$ ½

$\Rightarrow \qquad PQ = PT + PT$

$\Rightarrow \qquad PQ = 2PT$

$\Rightarrow \qquad \frac{1}{2} PQ = PT$

Hence, the common tangent to the circle at C, bisects the common tangents at P and Q. **1**

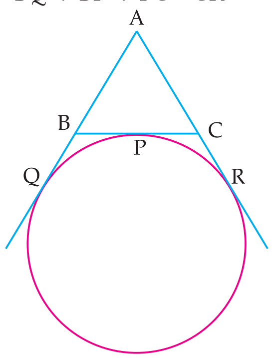
AI **Q. 3. If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that $AQ = \frac{1}{2}(BC + CA + AB)$.**

A **[CBSE OD Set-I, 2020]**

Sol. $BC + CA + AB$

$= (BP + PC) + (AR - CR) + (AQ - BQ)$ ½

$= AQ + AR - BQ + BP + PC - CR$



1

$\because$ From the same external point, the tangent segments drawn to a circle are equal. ½

From the point B, $BQ = BP$

From the point A, $AQ = AR$

From the point C, $CP = CR$

$\therefore$ Perimeter of ΔABC, *i.e.*,

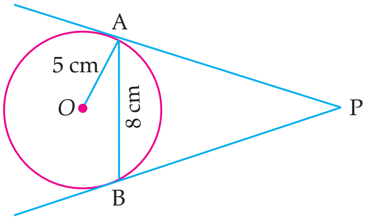
$$AB + BC + CA = 2AQ - BQ + BQ + CR - CR]$$

$\Rightarrow \qquad 2AQ = AB + BC + CA$

$\Rightarrow \qquad AQ = \frac{1}{2}(BC + CA + AB)$

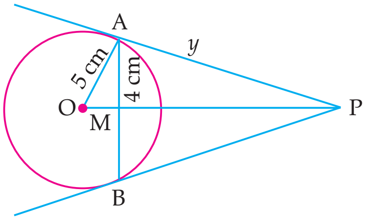
Hence proved. 1

Q. 4. In figure AB is a chord of length 8 cm of a circle of radius 5 cm. The tangents to the circle at A and B intersect at P. Find the length of AP.



A **[CBSE Delhi Set-I, 2019, Compt. Set I, II, III 2018]**

Sol.



Given, $AB = 8$ cm $\Rightarrow AM = 4$ cm.

$\therefore \qquad OM = \sqrt{OA^2 - AM^2}$

(By using Pythagoras theorem)

$$OM = \sqrt{5^2 - 4^2} = 3 \text{ cm.}$$

Let $AP = y$ cm, $PM = x$ cm.

$\therefore$ ΔOAP is a right angle triangle.

$\therefore \qquad OP^2 = OA^2 + AP^2$

(Again using Pythagoras theorem)

$$(x + 3)^2 = y^2 + 25$$

$\Rightarrow \qquad x^2 + 9 + 6x = y^2 + 25$ ...(i) ½

Also, $\qquad x^2 + 4^2 = y^2$ ...(ii) ½

$$x^2 + 6x + 9 = x^2 + 16 + 25$$

$$6x = 32$$

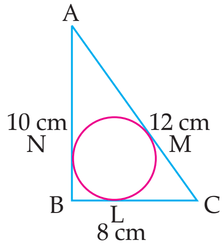
$\Rightarrow \qquad x = \frac{32}{6}$ *i.e.*, $\frac{16}{3}$ cm

$$y^2 = x^2 + 16 = \frac{256}{9} + 16$$

$= \frac{400}{9}$ **1**

$y = \frac{20}{3}$ cm or $6\frac{2}{3}$ cm. **1**

AI **Q. 5. In the given figure a circle is inscribed in a ΔABC having sides BC = 8 cm, AB = 10 cm and AC = 12 cm. Find the length BL, CM and AN.**



[A] [CBSE Delhi Set-II, 2019]
[Delhi Set-I, II, III, 2016]

Sol. Let $BL = x = BN$

(Tangent from external point B)

$\therefore$ $CL = 8 - x = CM$

(Tangent from external point C)

$\because$ $AC = 12$

$\Rightarrow$ $AM = 4 + x = AN$ **1**

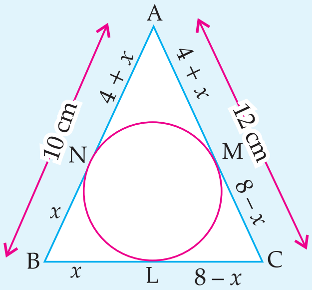
(Tangent from external point A)

Now $AB = AN + NB = 10$

$\Rightarrow$ $x + 4 + x = 10$

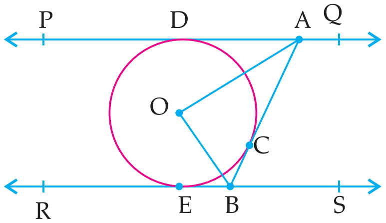
$\Rightarrow$ $x = 3$ **1**

$\therefore$ $BL = 3$ cm, $CM = 5$ cm and $AN = 7$ cm. **1**



Q. 6. **In Figure PQ and RS are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting PQ at A and RS at B. Prove that ∠AOB = 90°.**

[CBSE Delhi Set-I, II, III, 2017]
[A] [CBSE OD Set-1, 2019]



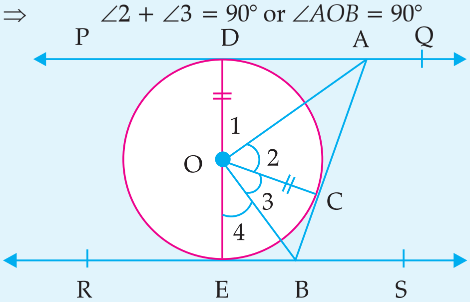
Sol. $\Delta AOD \cong \Delta AOC$ [SAS] **1**

$\Rightarrow$ $\angle 1 = \angle 2$ ½

Similarly $\angle 4 = \angle 3$ ½

$\Rightarrow$ $\angle 1 + \angle 4 = \angle 2 + \angle 3 = \frac{1}{2}(180°)$

$\Rightarrow$ $\angle 2 + \angle 3 = 90°$ or $\angle AOB = 90°$ ½



½

Alternate method :

$\Delta OAD \cong \Delta AOC$ (By SAS)

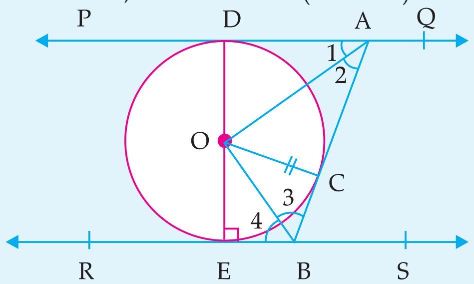
$\Rightarrow$ $\angle 1 = \angle 2$ **1**

Similarly, $\angle 4 = \angle 3$ ½

But $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$ ($\because PQ \,||\, RS$)

$\Rightarrow$ $\angle 2 + \angle 3 = \angle 1 + \angle 4 = \frac{1}{2}(180°) = 90°$

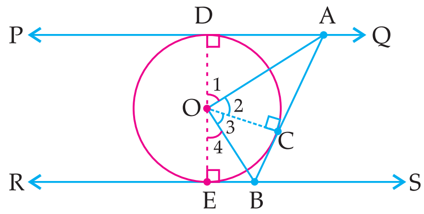
$\therefore$ In ΔAOB, $\angle AOB = 180° - (\angle 2 + \angle 3) = 90°$ ½



½

[CBSE Marking Scheme, 2019]

Detailed Solution:



½

In Δ DOA and Δ COA

$DA = AC$

(Tangents drawn from common point)

$\angle ODA = \angle OCA = 90°$

(angle between tangent and radius)

$OD = OC$ (radius of circle)

$\therefore$ $\Delta\, DOA \cong \Delta COA$ (By SAS) ½

Hence, $\angle 1 = \angle 2$ *i.e.*, $\angle DOA = \angle COA$ (By cpct) ...(i)

Similarly,

$\Delta\, BOC \cong \Delta\, BOE$ (By SAS) ½

$\therefore$ $\angle 3 = \angle 4$ *i.e.*, $\angle COB = \angle BOE$ (By cpct) ...(ii)

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$ ½

(angles on a straight line)

$2\angle 2 + 2\angle 3 = 180°$

[from eq. (i) & eq. (ii)]

$\angle 2 + \angle 3 = 90°$

i.e., $\angle AOC + \angle BOC = 90°$

or $\angle AOB = 90°$ **Hence Proved. 1**

COMMONLY MADE ERROR

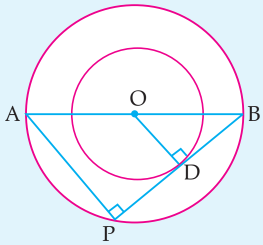
- Some candidates could not apply the appropriate theorem to find out the unknown angles.

ANSWERING TIP

- Learn circle and related angle properties, cyclic properties, tangent and secant properties thoroughly.

Q. 7. **The radii of two concentric circles are 13 cm and 8 cm. AB is a diameter of the bigger circle and BD is a tangent to the smaller circle touching it at D and intersecting the larger circle at P on producing. Find the length of AP.** U [CBSE SQP, 2018-19]

Sol.



1

$\angle APB = 90°$ (angle in semi-circle)

and $\angle ODB = 90°$ (radius is perpendicular to tangent)

$$\Delta ABP \sim \Delta OBD$$

$$\Rightarrow \quad \frac{AB}{OB} = \frac{AP}{OD} \qquad 1$$

$$\Rightarrow \quad \frac{26}{13} = \frac{AP}{8}$$

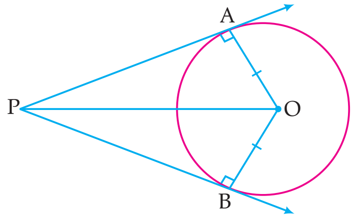
Hence, $AP = 16$ cm 1

[CBSE Marking Scheme, 2018-19]

Q. 8. **Prove that the lengths of two tangents drawn from an external point to a circle are equal.**

A [CBSE OD Set-I, II, III, 2018]

Sol. Given: AP and BP are tangents of circle having centre O. ½



½

To Prove : $AP = BP$ ½

Construction : Join OP, AO and BO ½

Proof : ΔOAP and ΔOBP

$OA = OB$ (Radius of circle)

$OP = OP$ (Common side)

$\angle OAP = \angle OBP = 90°$

(Radius – tangent angle)

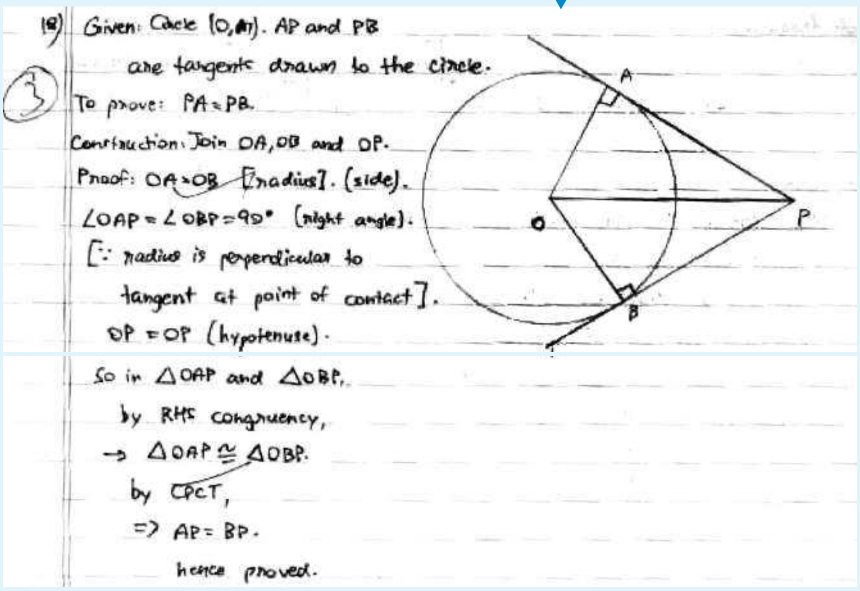
$\Delta OAP = \Delta OBP$ (RHS congruency rule)

$AP = BP$ (By cpct) **1**

Hence Proved.

Detailed Solution:

Topper Answer, 2018



18) Given: Circle (O, r). AP and PB are tangents drawn to the circle.

To prove: PA = PB.

Construction: Join OA, OB and OP.

Proof: OA = OB [radius]. (side).

∠OAP = ∠OBP = 90° (right angle).

[∵ radius is perpendicular to tangent at point of contact].

OP = OP (hypotenuse).

So in △OAP and △OBP,

by RHS congruency,

→ △OAP ≅ △OBP.

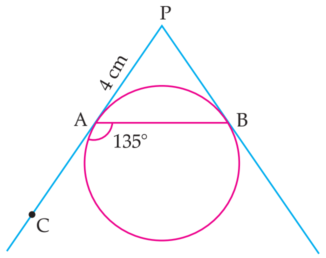
by CPCT,

=> AP = BP.

hence proved.

3

Q. 9. **In the given figure, PA and PB are tangents to a circle from an external point P such that** PA **= 4 cm and** $\angle BAC$ **= 135°. Find the length of chord** AB**.**



U [CBSE OD Set I, II, III, 2017]

Sol. $PA = PB = 4$ cm

(Tangents from external point) ½

$\angle PAB = 180° - 135° = 45°$

(Supplementary angles)

$\angle ABP = \angle PAB = 45°$

(Opposite angles of equal sides) ½

$\therefore \quad \angle APB = 180° - 45° - 45°$

$= 90°$

So, ΔABP is an isosceles right angled triangle.

$\Rightarrow \quad AB^2 = 2AP^2$ **1**

$\Rightarrow \quad AB^2 = 32$ **1**

Hence, $\quad AB = \sqrt{32} = 4\sqrt{2}$ cm

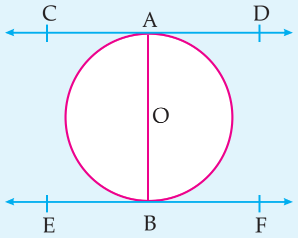
[CBSE Marking Scheme, 2017]

Q. 10. Prove that the tangents drawn at the ends of the diameter of a circle are parallel.

A **[CBSE Delhi Set I, II, III, 2017]**

Sol. Let AB be the diameter of a given circle and let CD and EF be the tangents drawn to the circle at A and B respectively.

$AB \perp CD$ and $AB \perp EF$ **1**



$\therefore \quad \angle CAB = 90°$ and $\angle ABF = 90°$ ½

$\angle CAB = \angle ABF$

and $\quad \angle ABE = \angle BAD$ ½

$\angle CAB$ and $\angle ABF$ also $\angle ABE$ and $\angle BAD$ are alternate interior angles. **1**

$\therefore \quad CD \parallel EF$ **Hence proved.**

[CBSE Marking Scheme, 2017]

Q. 11. ABC is a triangle. A circle touches sides AB and AC produced and side BC at X, Y and Z respectively. Show that

$AX = \frac{1}{2}$ **perimeter of** ΔABC.

A **[Board Term-2, 2016]**

Sol. See Q.3. from SATQ-II.

Long Answer Type Questions — 5 marks each

Q. 1. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

A **[CBSE Delhi Region, 2019]**
[Foreign Set-I, II, III, 2017]

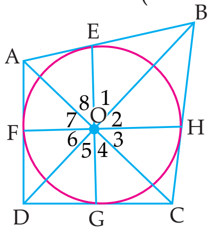
Sol. Given: A circle with centre O is inscribed in a quadrilateral $ABCD$.

In ΔAEO and ΔAFO,

$OE = OF$ (radii of circle)

$\angle OEA = \angle OFA = 90°$

(radius is $\perp^r$ to tangent) **1**



1

The point of contact is perpendicular to the tangent.

$OA = OA$ (common side)

$\Delta AEO \cong \Delta AFO$

(R.H.S. congruency)

$\angle 7 = \angle 8$ (By cpct) ...(i) **1**

Similarly,

$\angle 1 = \angle 2$ ...(ii)

$\angle 3 = \angle 4$ ...(iii)

$\angle 5 = \angle 6$ ...(iv)

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$ **1**

(angle around a point is 360°)

$2\angle 1 + 2\angle 8 + 2\angle 4 + 2\angle 5 = 360°$

$\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180°$

$(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180°$ **1**

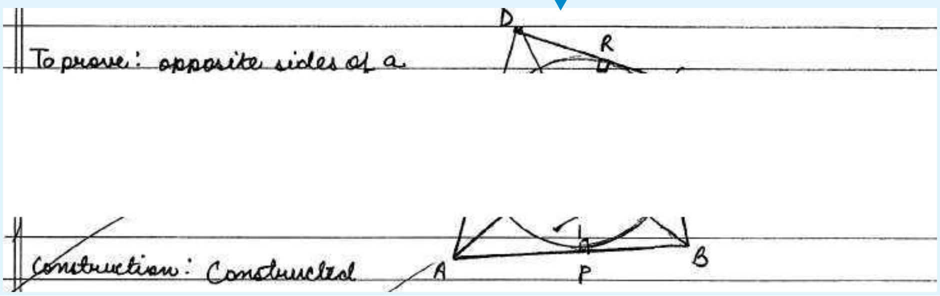
$\angle AOB + \angle COD = 180°$

Hence Proved.

Alternate Method:

Topper Answer, 2018

Sol.


To prove: opposite sides of a

Construction: Constructed

a quadrilateral ABCD, circumscribing a circle (centre O).
Circle touches AB, BC, CD, DA at P, Q, R, S respectively.

To prove : $\angle AOB + \angle COD = 180°$
Or $\angle AOD + \angle BOC = 180°$

We know, that tangents from same exterior point subtend equal angle at the centre of circle with radius.

$\therefore \angle AOP = \angle AOS = \angle 1$ (say)

Similarly, $\angle BOP = \angle BOQ = \angle 2$
$\angle COQ = \angle COR = \angle 3$
$\angle DOR = \angle \angle DOS = \angle 4.$

$\therefore \angle AOP + \angle BOP + \angle BOQ + \angle COQ + \angle COR + \angle DOR + \angle DOS + \angle AOS = 360°$
[Complete angle around a point]

$\Rightarrow 2\angle 1 + 2\angle 2 + 2\angle 3 + 2\angle 4 = 360°$
$\Rightarrow \angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$
$\Rightarrow (\angle 1 + \angle 2) + (\angle 3 + \angle 4) = 180°.$
$\Rightarrow \angle AOB + \angle COD = 180°$

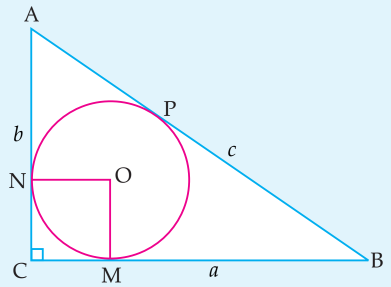
Or $(\angle 1 + \angle 4) + (\angle 2 + \angle 3) = 180°$
$\Rightarrow \angle AOD + \angle BOC = 180°$
Hence, proved !

5

Q. 2. ***a*, *b* and *c* are the sides of a right triangle, where *c* is the hypotenuse. A circle, of radius *r*, touches the sides of the triangle. Prove that $r = \dfrac{a+b-c}{2}$.**

A **[CBSE SA-2, 2016]**

Sol.



1

Let circle touches *CB* at *M*, *CA* at *N* and *AB* at *P*.
Now $OM \perp CB$ and $ON \perp AC$
(radius ⊥ tangent)

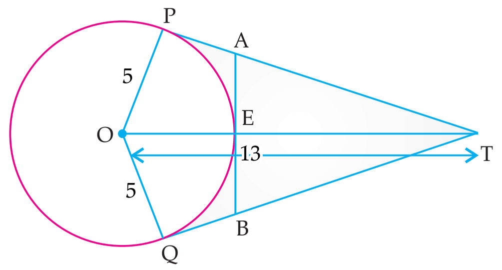
$OM = ON$ (radii)
$CM = CN$ (Tangents) **1**

$\therefore$ *OMCN* is a square.
Let $OM = r = CM = CN$ **1**
$AN = AP$, $CN = CM$ and $BM = BP$
(tangent from external point)

$AN = AP$
$\Rightarrow AC - CN = AB - BP$ **1**
$b - r = c - BM$
$b - r = c - (a - r)$
$b - r = c - a + r$
$\therefore 2r = a + b - c$
$r = \dfrac{a+b-c}{2}.$ **1**

Hence Proved.
[CBSE Marking Scheme, 2016]

Q. 3. In Fig. O is the centre of a circle of radius 5 cm. T is a point such that *OT* = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



U **[CBSE Delhi Set I, II, III, 2016]**

Sol. $PT = \sqrt{169 - 25} = 12$ cm

and $TE = OT - OE = 13 - 5$

$= 8$ cm ½ + ½

Let $PA = AE = x$. (Tangents)

Then, $TA^2 = TE^2 + EA^2$ **1**

or, $(12 - x)^2 = 8^2 + x^2$ **1**

$24x = 80$

or, $x = 3.3$ cm. (Approx.) **1**

Thus $AB = 2 \times x = 2 \times 3{\cdot}3$

$= 6.6$ cm. (Approx.) **1**

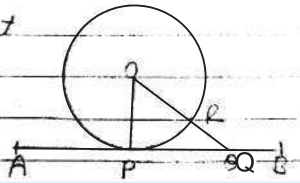
[CBSE Marking Scheme, 2016]

Q. 4. Prove that tangent drawn at any point of a circle is perpendicular to the radius through the point of contact.

[CBSE OD Set II, 2016]

Topper Answer, 2016

Sol. Given – A circle C(O, OP) and tangent at P.



To prove – OP ⊥ PQ

Constⁿ – Extend OR to Q, at AB

Proof – We have –

OP = OR (radius)

OQ = OR + RQ

Clearly OQ > OR

∴ OQ > OP

The shortest line joining a point to any point on given line is ⊥r to that line

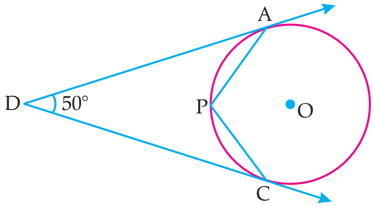
⇒ OP ⊥ AB

or OP ⊥ PQ

Hence proved

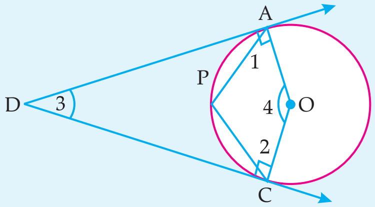
5

Q. 5. In the given figure, O is the centre of the circle. Determine ∠*APC*, if DA and DC are tangents and ∠*ADC* = 50°.



A **[Board Term-2, 2015]**

Sol.



1

Given *DA* and *DC* are tangents from point *D* to a circle with centre *O*.

$\angle 1 = \angle 2 = 90°$

(radius ⊥ tangent) **1**

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$ **1**

or, $90° + 90° + 50° + \angle 4 = 360°$

or, $\angle 4 = 130°$

∴ Reflex $\angle 4 = 360° - 130° = 230°$ **1**

$\angle APC = \frac{1}{2}$ reflex $\angle 4$

(angle subtended at centre)

$\angle APC = \frac{1}{2} \times 230° = 115°$ **1**

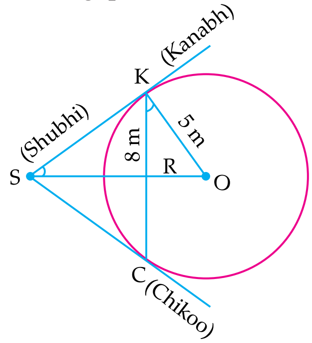
[CBSE Marking Scheme, 2015]

Visual Case Based Questions

4 marks each

Note: Attempt any four sub parts from each question. Each sub part carries 1 mark

AI **Q. 1. There is a circular filed of radius 5 m. Kanabh, Chikoo and Shubhi are playing with ball, in which Kanabh and Chikoo are standing on the boundary of the circle. The distance between Kanabh and Chikoo is 8 m. From Shubhi point S, two tangents are drawn as shown in the figure. Give the answer of the following questions.** C + AE



(i) What is the relation between the lengths of SK and SC ?

(a) SK ≠ SC (b) SK = SC

(c) SK > SC (d) SK < SC.

Sol. Correct Option: **(b)**

Explanation: We know that the lengths of tangents drawn from an external point to a circle are equal. So SK and SC are tangents to a circle with centre O.

$\therefore \quad SK = SC$ **1**

(ii) The length (distance) of OR is:

(a) 3 m (b) 4 m

(c) 5 m (d) 6 m.

Sol. Correct Option: **(a)**

Explanation: In question 1, we have proved

$SK = SC$

Then ΔSKC is an isosceles triangle and SO is the angle bisector of ∠KSC. So, OS ⊥ KC.

∴ OS bisects KC, gives KR = RC = 4 cm.

Now, $OR = \sqrt{OK^2 - KR^2}$

(By using Pythagoras theorem)

$= \sqrt{5^2 - 4^2} = \sqrt{25 - 16}$

$= \sqrt{25 - 16}$

$= \sqrt{9}$

$= 3$ m. **1**

(iii) The sum of angles SKR and OKR is:

(a) 45° (b) 30°

(c) 90° (d) none of these

Sol. Correct Option: **(c)**

Explanation:

$\angle SKR + \angle OKR = \angle OKR$

$= 90°$ (Radius is ⊥^{r} to tangent) **1**

(iv) The distance between Kanabh and Shubhi is:

(a) $\frac{10}{3}$ m (b) $\frac{13}{3}$ m

(c) $\frac{16}{3}$ m (d) $\frac{20}{3}$ m

Sol. Correct Option: **(d)**

Explanation: ΔSKR and ΔRKO,

$\angle RKO = \angle KSR$

and $\angle SRK = \angle ORK$

$\therefore \quad \Delta KSR \sim \Delta OKR$ (By AA Similarity)

Then $\frac{SK}{KO} = \frac{RK}{RO}$

$\Rightarrow \frac{SK}{5} = \frac{4}{3}$

(RO = 3 m, proved in Q.2.)

$\Rightarrow 3SK = 20$

$\Rightarrow SK = \frac{20}{3}$

Hence, the distance between Kanabh and Shubhi is $\frac{20}{3}$ m. **1**

(v) What is the mathematical concept related to this question ?

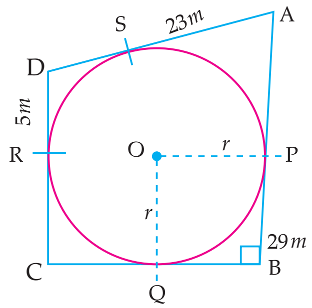
(a) Constructions (b) Area

(c) Circle (d) none of these

Sol. Correct Option: **(c)**

Explanation: The mathematical concept (**Circle**) is related to this question. **1**

AI **Q. 2. ABCD is a playground. Inside the playground a circular track is present such that it touches AB at point P, BC at Q, CD at R and DA at S.**



See the above figure and give answer of the following questions: C + AE

(i) If $DR = 5$ m, then DS is equal to:

(a) 6 m (b) 11 m

(c) 5 m (d) 18 m

Sol. Correct Option: **(c)**

Explanation:

$DR = 5$ m (given)

$\therefore$ $DR = DS$

(Length of tangents are equal)

i.e., $DS = 5$ m. **1**

(ii) The length of AS is:

(a) 18 m **(b) 13 m**

(c) 14 m **(d) 12 m**

Sol. Correct Option: **(a)**

Explanation: We have $AD = 23$ m.

and $DS = 5$ m (Proved in Q.1)

$\therefore$ $AS = AD - DS$

$= (23 - 5)$ m $= 18$ m. **1**

(iii) The length of PB is:

(a) 12 m **(b) 11 m**

(c) 13 m **(d) 20 m**

Sol. Correct Option: **(b)**

Explanation: We have,

$AB = 29$ m

But $AS = $ AP (lengths of tangents are equal)

and $AS = 18$ m (Proved in Q.2)

$\therefore$ $AP = 18$ m

Now, $PB = AB - AP$

$= (29 - 18)$ m

$= 11$ m. **1**

(iv) What is the angle of OQB?

(a) 60° **(b) 30°**

(c) 45° **(d) 90°**

Sol. Correct Option: **(d)**

Explanation: $\angle OQB = 90°$ (Radius is $\perp^r$ to tangent) **1**

(v) What is the diameter of given circle?

(a) 22 m **(b) 33 m**

(c) 20 m **(d) 30 m**

Sol. Correct Option: **(a)**

Explanation:

$\because$ $PB = 11$ m (proved in Q.3)

But $PB = BQ$ (lengths of tangents are equal)

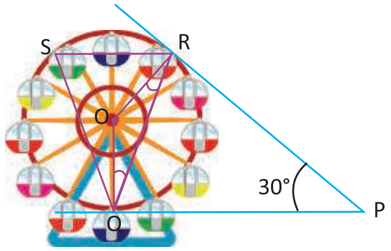
$\therefore$ $BQ = 11$ m

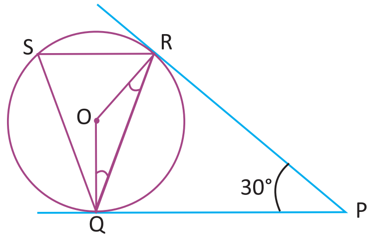
or $r = OQ = QB = 11$ m

Hence, diameter $= 2r = 2 \times 11 = 22$ m. **1**

Q. 3. A Ferris wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passenger-carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas, or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity.

After taking a ride in Ferris wheel, Aarti came out from the crowd and was observing her friends who were enjoying the ride. She was curious about the different angles and measures that the wheel will form. She forms the figure as given below.





(i) In the given figure find $\angle$ROQ.

(a) 60 **(b) 100**

(c) 150 **(d) 90**

Sol. Correct option: **(c).**

(ii) Find $\angle$RQP.

(a) 75 **(b) 60**

(c) 30 **(d) 90**

Sol. Correct option: **(a).**

(iii) Find $\angle$RSQ.

(a) 60 **(b) 75**

(c) 100 **(d) 30**

Sol. Correct option: **(b).**

(iv) Find $\angle$ORP.

(a) 90 **(b) 70**

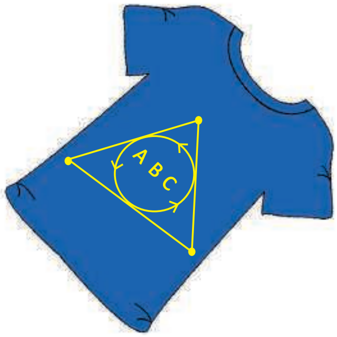
(c) 100 **(d) 60**

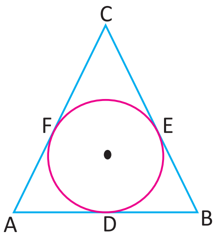
Sol. Correct option: **(a).**

Explanation: $\angle ORP = 90°$

Because, radius of circle is perpendicular to tangent.

Q. 4. Varun has been selected by his School to design logo for Sports Day T-shirts for students and staff. The logo design is as given in the figure and he is working on the fonts and different colours according to the theme. In given figure, a circle with centre O is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. The lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively.





(i) Find the length of AD.

(a) 7 **(b)** 8

(c) 5 **(d)** 9

Sol. Correct option: **(a).**

(ii) Find the Length of BE.

(a) 8 **(b)** 5

(c) 2 **(d)** 9

Sol. Correct option: **(b).**

(iii) Find the length of CF.

(a) 9 **(b)** 5

(c) 2 **(d)** 3

Sol. Correct option: **(d).**

(iv) If radius of the circle is 4 cm, find the area of ΔOAB.

(a) 20 **(b)** 36

(c) 24 **(d)** 48

Sol. Correct option: **(c).**

(v) Find area of ΔABC

(a) 50 **(b)** 60

(c) 100 **(d)** 90

Sol. Correct option: **(b).**